Colour superconductivity in a strong magnetic field

2006 J. Phys. A: Math. Gen. 39 6349

(http://iopscience.iop.org/0305-4470/39/21/S27)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 6349–6355 doi:10.1088/0305-4470/39/21/S27

Colour superconductivity in a strong magnetic field

Efrain J Ferrer[1], **Vivian de la Incera**[1] **and Cristina Manuel**[2]

[1] Department of Physics, Western Illinois University, Macomb, IL 61455, USA
[2] Instituto de Fisica Corpuscular, CSIC-U. de Valencia, 46071 Valencia, Spain

E-mail: EJ-Ferrer@wiu.edu, V-Incera@wiu.edu and Cristina.Manuel@ific.uv.es

Received 30 November 2005, in final form 12 January 2006
Published 10 May 2006
Online at stacks.iop.org/JPhysA/39/6349

Abstract
We explore the effects of an applied strong external magnetic field in a three flavour massless colour superconductor. The long-range component of the B field that penetrates the superconductor enhances some quark condensates, leading to a different condensation pattern. The external field also reduces the flavour symmetries in the system, and thus it changes drastically the corresponding low energy physics. Our considerations are relevant for the study of highly magnetized compact stars.

PACS numbers: 12.38.Aw, 24.85.+p, 26.60.+c

1. Introduction

This workshop is devoted to study the influence of extreme conditions on a quantum field theory. In this talk, we will particularly focus our attention to the study of quantum chromodynamics (QCD) under the conditions of high baryonic density and very strong magnetic fields.

After the discovery of the property of asymptotic freedom of QCD, it was soon realized that the behaviour of the theory in the situations of very high temperature and/or baryonic density would be drastically different than in vacuum. In those circumstances quarks and gluons should behave as almost free particles, as their coupling becomes very weak at high energy scales. Currently, the physics community is devoting many efforts to test these ideas experimentally [1, 2].

In different astrophysical settings, it is believed that the density is so high that the hadrons melt into their fundamental constituents, giving rise to quark matter. It has been known for long time now that cold dense quark matter should exhibit the phenomenon of colour superconductivity [3, 4]. It is our aim here to explain how a strong magnetic field affects this phenomenon. This is not a simple academic question. The real fact is that almost all compact stars sustain a strong magnetic field, of the order of $B \sim 10^{12}$–10^{14} G for pulsars, and of $B \sim 10^{14}$–10^{15} G for magnetars. A comparison of the gravitational and magnetic energies of a compact star tells us that the maximum fields may be as high as $B \sim 10^{18} - 10^{19}$ G. The common belief is that all the above-mentioned compact objects are neutron stars, where neutrons are in a superfluid phase, while the protons are in a superconducting one, probably of type I. In [5] it was found that an external magnetic field influences both quantitatively and qualitatively the colour superconducting state. We will explain here why an applied external magnetic field has different effects in an electromagnetic or in a colour superconductor. These considerations allow us to state that if quark matter in a colour superconducting state is realized in any of those compact objects, the dynamics of the associated magnetic field should be drastically different than in neutron matter. We hope to explore the astrophysical consequences of our work in the near future.

2. Electromagnetic and colour superconductivity

Our present microscopic understanding of the phenomenon of colour superconductivity relies on techniques of BCS theory adapted to dense quark matter.

The seminal work of Bardeen, Cooper and Schrieffer gave the microscopic explanation of the electromagnetic superconductivity found in some metals at low temperature T. The main ingredients of BCS theory are the following: a finite density of fermions at low T, occupying a Fermi sea, and an attractive interaction between them occurring close to the Fermi surface. Even if the interaction is very weak, this will render the existing ground state unstable favouring the formation of Cooper pairs of fermions. In a metal at low temperature the attractive interaction is mediated through phonons, the quantum of vibrational energy of the metal.

In electromagnetic superconductivity the Cooper pair, having two electrons (or any other two charged fermions), has a net electric charge. The gauge symmetry is spontaneously broken, and the photon acquires a mass through the Anderson–Higgs mechanism. A weak magnetic field is thus screened in the superconductor: this is the Meissner effect. However, a sufficiently strong magnetic field destroys the superconducting state. One intuitive way to understand why this is so is provided in the lower section of figure 1. The electrons in the Cooper pair have equal charges and opposite spins thus antiparallel magnetic moments. The field effect will be to align the two magnetic moments parallel to each other, therefore tending to break the superconducting bound state. Electromagnetic superconductivity and magnetism are thus anathema.

In high density QCD quarks fill out the states up to the Fermi surface—the first required ingredient of BCS theory. The fundamental interaction between two quarks mediated by one-gluon exchange has an attractive component in the colour antitriplet channel—the second required ingredient in BCS theory. At very large density the energy of the quarks at the Fermi surface will be large, so the attractive interaction between them will be weak. According to BCS theory even an arbitrarily weak interaction will do the trick of restructuring the ground state through the formation of Cooper pairs of quarks. Because the quarks carry ‘colour’ charge, the quark–quark pairs will carry nonzero colour charge too, and in general one may expect that some or all gluons may get a Meissner mass.

Quarks carry different quantum numbers (spin, colour, flavour). Who pairs with who? The answer depends on different QCD parameters, such as the quark masses and chemical potentials. The latter are further constrained by neutrality conditions. Then one talks about different colour superconducting phases, which are characterized by the different local and global symmetries which are spontaneously broken. In our analysis to study the influence of magnetism in the quark dynamics, and as the first approach to the problem, we will neglect

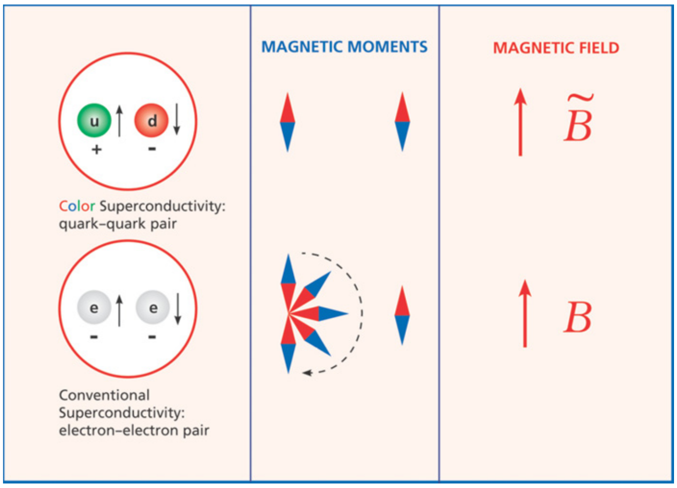

Figure 1. The picture provides an intuitive way to understand the magnetic reinforcement of colour superconductivity.

(This figure is in colour only in the electronic version)

quark mass effects, and all subsequent complications, which we hope to address in future studies.

Quarks, apart from colour, also carry an electromagnetic charge. Should then one expect the colour superconductor to also be an electromagnetic superconductor? The answer to this question requires a closer look into the quark superconducting state. As we will explain below, magnetism and colour superconductivity are on good terms.

3. Colour–flavour locking phase

The ground state of QCD at high baryonic density with three light quark flavours is described by the (spin-zero) condensates [6]

$$\langle q_L^{ai} q_L^{bj} \rangle = -\langle q_R^{ai} q_R^{bj} \rangle = \Delta_A \epsilon^{abc} \epsilon_{ijc}, \tag{1}$$

where $q_{L/R}$ are Weyl spinors (a sum over spinor indices is understood), and a, b and i, j denote flavour and colour indices, respectively. For simplicity we have neglected a colour sextet component of the condensate, as it is a subleading effect.

The diquark condensates lock the colour and flavour transformations breaking both—thus the name colour–flavour locked (CFL) phase. The symmetry breaking pattern in the CFL phase is $SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{C+L+R}$. There are only nine Goldstone bosons that survive to the Anderson–Higgs mechanism. One is a singlet, scalar mode, associated with the breaking of the baryonic symmetry, and the remaining octet is associated with the axial $SU(3)_A$ group, just like the octet of mesons in vacuum. At sufficiently high density, the anomaly is suppressed, and then one can as well consider the spontaneous breaking of an approximated $U(1)_A$ symmetry, and an additional pseudo Goldstone boson.

In the CFL phase the eight gluons become massive and do not form gluon condensates, as in vacuum. The octets of gluons and Goldstone bosons in the CFL phase are analogous to the octets of vector and light pseudoscalar mesons in vacuum, respectively. This fact has led Schafer and Wilczek to the conjecture that there might exist some sort of continuity between the properties of QCD at low and high densities [7].

An important feature of spin-zero colour superconductivity is that although the colour condensate has nonzero electric charge, there is a linear combination of the photon A_μ and a gluon G^8_μ that remains massless [6, 8],

$$\widetilde{A}_\mu = \cos\theta A_\mu - \sin\theta G^8_\mu, \tag{2}$$

while the orthogonal combination $\widetilde{G}^8_\mu = \sin\theta A_\mu + \cos\theta G^8_\mu$ is massive. In the CFL phase the mixing angle θ is sufficiently small ($\sin\theta \sim e/g \sim 1/40$). Thus, the penetrating field in the colour superconductor is mostly formed by the photon with only a small gluon admixture.

The unbroken $U(1)$ group corresponding to the long-range rotated photon (i.e., the $\widetilde{U}(1)_{\text{e.m.}}$) is generated, in flavour–colour space, by $\widetilde{Q} = Q \times 1 - 1 \times Q$, where Q is the electromagnetic charge generator. We use the convention $Q = -\lambda_8/\sqrt{3}$, where λ_8 is the eighth Gell–Mann matrix. Thus our flavour-space ordering is (s, d, u). In the nine-dimensional flavour–colour representation that we will use here (the colour indexes we are using are (1,2,3)=(b,g,r)), the $\widetilde{Q}$ charges of the different quarks, in units of $\widetilde{e} = e\cos\theta$, are

s_1	s_2	s_3	d_1	d_2	d_3	u_1	u_2	u_3
0	0	–	0	0	–	+	+	0

(3)

While a weak magnetic field only changes slightly the properties of the CFL superconductor, in the presence of a strong magnetic field the condensation pattern is changed, giving rise to a new phase—the magnetic colour–flavour locked (MCFL) phase.

4. Magnetic colour–flavour locking phase

An external magnetic field to the colour superconductor will be able to penetrate it in the form of a 'rotated' magnetic field $\widetilde{B}$. With respect to this long-ranged field, although all the superconducting pairs are neutral, a subset of them are formed by quarks with opposite rotated $\widetilde{Q}$ charges. Hence, it is natural to expect that these kinds of condensates will be affected by the penetrating field, as the quarks couple minimally to the rotated gauge field. Furthermore, one may expect that these condensates are strengthened by the penetrating field, since their paired quarks, having opposite $\widetilde{Q}$-charges and opposite spins, have parallel (instead of antiparallel) magnetic moments (see the upper section of figure 1). The situation here has some resemblance to the magnetic catalysis of chiral symmetry breaking [9], in the sense that the magnetic field strengthens the pair formation. Despite this similarity the way the field influences the pairing mechanism in the two cases is quite different, as we will discuss later on.

A strong magnetic field affects the flavour symmetries of QCD, as different quark flavours have different electromagnetic charges. For three light quark flavours, only the subgroup of $SU(3)_L \times SU(3)_R$ that commutes with Q, the electromagnetic generator, is a symmetry of the theory. Equally, in the CFL phase a strong $\widetilde{B}$ field should affect the symmetries in the theory, as $\widetilde{Q}$ does not commute with the whole locked $SU(3)$ group. Based on this consideration, we proposed the following diquark (spin-zero) condensate, [5]

$$\langle q^{ai}_L q^{bj}_L \rangle = -\langle q^{ai}_R q^{bj}_R \rangle = \Delta_A \epsilon^{ab3}\epsilon_{ij3} + \Delta^B_A(\epsilon^{ab2}\epsilon_{ij2} + \epsilon^{ab1}\epsilon_{ij1}), \tag{4}$$

and as for the CFL case, we have only considered the leading antitriplet colour channel. For a discussion of the remaining allowed structures in the subleading sextet channel, see [5]. Here we have been guided by the principle of highest symmetry, that is, the pair condensation should retain the highest permitted degree of symmetry, as then quarks of different colours and flavours will participate in the condensation process to guarantee a maximal attractive channel at the Fermi surface [6].

In the MCFL phase the symmetry breaking pattern is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)^{(1)}_A \times U(1)_B \times U(1)_{\rm e.m.} \to SU(2)_{C+L+R} \times \widetilde{U}(1)_{\rm e.m.}$. Here the symmetry group $U(1)^{(1)}_A$ is related to a current which is an anomaly free linear combination of u, d and s axial currents, and such that $U(1)^{(1)}_A \subset SU(3)_A$. The locked $SU(2)$ group corresponds to the maximal unbroken symmetry, such that it maximizes the condensation energy. The counting of broken generators, after taking into account the Anderson–Higgs mechanism, tells us that there are only five Goldstone bosons. As in the CFL case, one is associated with the breaking of the baryon symmetry; three Goldstone bosons are associated with the breaking of $SU(2)_A$, and the other one is associated with the breaking of $U(1)^{(1)}_A$. If the effects of the anomaly could be neglected, there would be another pseudo Goldstone boson associated with the $U(1)_A$ symmetry.

To study the MCFL phase we used a Nambu–Jona–Lasinio (NJL) four-fermion interaction abstracted from one-gluon exchange [6]. This simplified treatment, although disregards the effect of the $\widetilde{B}$-field on the gluon dynamics and assumes the same NJL couplings for the system with and without the magnetic field, keeps the main attributes of the theory, providing the correct qualitative physics. The NJL model is treated as the proper effective field theory to study colour superconductivity in the regime of moderate densities. The model is defined by two parameters, a coupling constant g and an ultraviolet cutoff Λ. The cutoff should be much higher than the typical energy scales in the system, that is, the chemical potential μ and the magnetic energy $\sqrt{\widetilde{e}\widetilde{B}}$.

The study of the MCFL gap equations presents several technical difficulties, which we will only briefly mention here. The computation of the field-dependent quark propagators is laborious, but it can be managed with the use of Ritus' method, originally developed for charged fermions [10] and recently extended to charged vector fields [11]. In Ritus' approach the diagonalization in momentum space of charged fermion Green's functions in the presence of a background magnetic field is carried out using the eigenfunction matrices $E_p(x)$. These are the wavefunctions of the asymptotic states of charged fermions in a uniform magnetic field and play the role in the magnetized medium of the usual plane-wave (Fourier) functions e^{ipx} at zero field. With the help of the $E_p(x)$ functions, one can compute the propagators in momentum space, which depend on a discrete index that labels the Landau levels. With these propagators, one can derive the MCFL gap equations.

The gap equations for an arbitrary value of the magnetic field are extremely difficult to solve, and they require a numerical treatment. However, we have found a situation where an analytical solution can be found. This corresponds to the case $\widetilde{e}\widetilde{B} > \mu^2/2$, where μ is the chemical potential. In this case, only charged quarks in the lowest Landau level contribute to the gap equation; a situation that drastically simplifies the analysis.

In BCS theory, and in the presence of contact interactions, the fermionic gap has an exponential dependence on the inverse of the density of states close to the Fermi surface, which is proportional to μ^2. Effectively, one can find that within the NJL model the CFL gap reads

$$\Delta_A^{\rm CFL} \sim 2\sqrt{\delta\mu}\,\exp\left(-\frac{3\Lambda^2\pi^2}{2g^2\mu^2}\right) \tag{5}$$

with $\delta \equiv \Lambda - \mu$. In the MCFL phase, when $\widetilde{e}\widetilde{B} > \mu^2/2$, we find instead

$$\Delta_A^B \sim 2\sqrt{\delta\mu}\,\exp\left(-\frac{3\Lambda^2\pi^2}{g^2(\mu^2+\widetilde{e}\widetilde{B})}\right) \tag{6}$$

For the value of the remaining gaps of the MCFL phase, see [5]. All the gaps feel the presence of the external magnetic field. As expected, the effect of the magnetic field in Δ_A^B is to increase the density of states, which enters the argument of the exponential as typical of a BCS solution. The density of states appearing in (6) is just the sum of those of neutral and charged particles participating in the given gap equation (for each Landau level, the density of states around the Fermi surface for a charged quark is $\widetilde{e}\widetilde{B}/2\pi^2$). The gap formed by $\widetilde{Q}$-neutral particles, although modified by the $\widetilde{B}$-field [5], has a subleading effect in the MCFL phase.

As mentioned at the beginning of this section, the situation here shares some similarities with the magnetic catalysis of chiral symmetry breaking [9]; however, the way the field influences the pairing mechanism in the two cases is quite different. The particles participating in the chiral condensate are near the surface of the Dirac sea. The effect of a magnetic field there is to effectively reduce the dimension of the particles at the lowest Landau level, which in turn strengthens their effective coupling, catalyzing the chiral condensate. Colour superconductivity, on the other hand, involves quarks near the Fermi surface, with a pairing dynamics that is already $(1 + 1)$ dimensional. Therefore, the $\widetilde{B}$-field does not yield further dimensional reduction of the pairing dynamics near the Fermi surface and hence the lowest Landau level does not have a special significance here. Nevertheless, the field increases the density of states of the $\widetilde{Q}$-charged quarks, and it is through this effect, as shown in (6), that the pairing of the charged particles is reinforced by the penetrating magnetic field.

5. Conclusions

We have presented the arguments to explain why three light flavour colour superconductivity is made stronger, not weaker, by the presence of magnetism. These arguments have been corroborated by an explicit computation of the quark gaps within a NJL model, in the regime of strong magnetic fields. To better understand the relevance of this new phase in astrophysics we need to explore the region of moderately strong magnetic fields $\widetilde{e}\widetilde{B} < \mu^2/2$, which requires us to carry out a numerical study of the gap equations including the effect of higher Landau levels.

The presence of a strong magnetic field affects the values of the quark gaps and thus, it will modify the equation of state of the colour superconductor, although we do not expect this to be a very pronounced effect. More drastically, the low energy physics of the MCFL phase would differ from that of the CFL phase, through the disappearance of light degrees of freedom. This fact will have consequences on several macroscopic properties of the superconductor, which we hope to explore soon.

6. Acknowledgments

The work of EJF and VI was supported in part by NSF grant no PHY-0070986, and CM was supported by MEC under grant no FPA2004-00996.

References

[1] Ludlam T 2005 *Nucl. Phys.* A **750** 9
[2] Weber F 2005 *Prog. Part. Nucl. Phys.* **54** 193
[3] Bailin D and Love A 1984 *Phys. Rep.* **107** 325
[4] Rajagopal K and Wilczek F The condensed matter physics of QCD *At the Frontier of Particle Physics / Handbook of QCD'* ed M Shifman (Singapore: World Scientific)
[5] Ferrer E J, de la Incera V and Manuel C 2005 *Phys. Rev. Lett.* **95** 152002
[6] Alford M, Rajagopal K and Wilczek F 1999 *Nucl. Phys.* B **537** 443
[7] Schafer T and Wilczek F 1999 *Phys. Rev. Lett.* **82** 3956
[8] Alford M, Berges J and Rajagopal K 2000 *Nucl. Phys.* B **571** 269

[9] Gusynin V P, Miransky V A and Shovkovy I A 1994 *Phys. Rev. Lett.* **73** 3499
Gusynin V P, Miransky V A and Shovkovy I A 1995 *Phys. Lett.* B **349** 477
Gusynin V P, Miransky V A and Shovkovy I A 1995 *Phys. Rev.* D **52** 4747
[10] Ritus V I 1972 *Ann. Phys., NY* **69** 555
Ritus V I 1978 *Sov. Phys.—JETP* **48** 788
[11] Elizalde E, Ferrer E J and de la Incera V 2002 *Ann. Phys., NY* **295** 33
Elizalde E, Ferrer E J and de la Incera V 2004 *Phys. Rev.* D **70** 043012